Effects of density-dependent quark mass on phase diagram of three-flavor quark matter

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Considering the density dependence of quark mass, we investigate the phase transition between the (unpaired) strange quark matter and the color-flavor-locked matter, which are supposed to be two candidates for the ground state of strongly interacting matter. We find that if the current mass of strange quark m_s is small, the strange quark matter remains stable unless the baryon density is very high. If m_s is large, the phase transition from the strange quark matter to the color-flavor-locked matter in particular to its gapless phase is found to be different from the results predicted by previous works. A complicated phase diagram of three-flavor quark matter is presented, in which the color-flavor-locked phase region is suppressed for moderate densities.

I. INTRODUCTION

The quark matter with three flavors (u, d and s) has been intensively studied for two decades. When the down quark chemical potential is larger than the strange quark mass, the strange quark matter (SQM) might be energetically favored with respect to two-flavor quark matter and even nuclear matter so that it should be the ground state of strongly interacting matter [1]. Within the framework of the bag model, Farhi and Jaffe pointed out that SQM with the strange quark mass $m_s < 140$MeV (and with appropriate bag constant) becomes the stable ground state for low baryon densities [2]. Based on this consideration, it is further speculated that some compact stars are made up not of neutrons but SQM, which are termed as strange quark stars [3]. On the other hand, the study of dense quark matter draws much attention due to the recent progress in understanding of color superconductivity. At high densities, the original color and flavor symmetries of three-flavor QCD, namely $SU(3)_{color} \times SU(3)_L \times SU(3)_R$, is suggested to be broken down to a diagonal subgroup $SU(3)_{color+L+R}$ via the Bardeen-Cooper-Schrieffer (BCS) pairing [4]. The three-flavor quark matter with the particular symmetry is called the color-flavor locked (CFL) matter and it is different from SQM which is the matter without the BCS pairing. As another state of strongly interacting matter, the CFL matter is widely believed to become " absolutely " stable for sufficiently high densities [5].

Thus there are two candidates for the ground state of three-flavor quark matter, CFL and SQM, which are stable for high and low densities respectively. The question is, in the moderate density region, which one of them is the ground state. In the other words, one concerns how SQM undergoes a phase transition to CFL with increase of density. Investigation on these issues is important for exploring the physics of strange

quark stars and/or the interior structure of compact stars. Ignoring the u and d quark masses, the CFL free energy takes the form

$$\Omega_{CFL} = -\frac{3\mu^4}{4\pi^2} + \frac{3m_s^2\mu^2}{4\pi^2} - \frac{3\Delta^2\mu^2}{\pi^2}, \qquad (1)$$

to order of m_s^2/μ, where μ is the quark chemical potential and Δ denotes the color superconducting gap. Using Eq.(1), Alford *et. al.* concluded that the CFL matter is more stable than the unpaired quark matter (exactly, SQM) as long as [6]

$$\mu \geq \frac{m_s^2}{4\Delta}. \qquad (2)$$

As the necessary condition for CFL existence [7], Eq.(2) is valid only for high densities. This inequality can not fully answer the question raised above because it does not address the phase structure at the moderate densities.

To illustrate this point more clearly, we draw the phase diagram based on Eq.(2) in the (m_s, μ) plane (Fig. 1)[1] . When the strange quark mass is small as $m_s < 175$MeV, Fig. 1 shows that SQM is excluded completely from the moderate density region $\mu = 0.3 - 1$GeV and the CFL matter including its gapless phase (gCFL [8]) dominates all over. However, for small m_s, SQM has been predicted to be the stable ground state [2] so that it should be favorable at least for low densities such as $\mu \sim 0.3$GeV. It implies that the relation determined by Eq.(2) is problematic more or less. If assuming that CFL emerges in strange quark stars, this contradiction becomes more obvious. Starting at very low density and increasing the matter density by increasing μ, the CFL formation must be preceded by presence of the stable SQM state. From this point of view, SQM remains stable for relatively low densities; otherwise, the self-bound quark stars could not exist and then the CFL formation would be impossible. Therefore, the moderate-density phase diagram shown in Fig. 1 needs to be reexamined, especially in the quark-star environment.

In fact, the implicit assumption for Fig. 1 is that only the current mass of strange quark m_s was considered in the descriptions for both CFL and SQM. According to the low-density QCD, the strange quark mass not merely originates from the explicit breaking of chiral symmetry. For low densities where SQM exists as the stable ground state, there is no reason to neglect the dynamical mass induced by the spontaneous chiral breaking. Once the dynamical mass is taken into account, it is found that the SQM stability window, e. g. the allowed region of the current mass m_s, is widened [9, 10]. This motivates us to reexamine both SQM and CFL while the dynamical mass is incorporated. In this work, we will introduce the density dependence of quark mass to investigate the phenomenological effects of the dynamical mass on the moderate-density phase diagram. This approach should be closer to reality and obviously helpful

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[1]Until now the actual value of Δ and its dependence on μ and m_s are not been well known, which are closely linked to the gap equation. In the literature, Δ was estimated to be of order tens to 100MeV. In Fig. 1 it is simply treated as a given parameter $\Delta \sim 25$MeV for moderate μ, say, $\mu \sim 0.5$GeV [8].

to clarify the problem of Fig. 1. In Sec.II, we briefly review the mass-density-dependent model [9] and consider the free energy of the CFL matter when the density-dependent quark mass is incorporated. We emphasize that the mechanism for existence of the gCFL phase needs to be reexamined in particular for relatively low densities. In Sec.III, we investigate the phase transitions from SQM to the conventional CFL phase and/or the gCFL phase and present a new phase diagram which is very different from Fig. 1.

II. THE MODEL

A. SQM and its stability

Following Ref.[9], the density-dependent quark mass is given by

$$m_D = C/(3\rho), \tag{3}$$

where ρ denotes the matter density and C is a model parameter which is constrained by the SQM stability conditions. If ignoring the current masses of u and d quarks, the masses for the light and strange quarks in this model are

$$M_u = M_d = m_D; \quad M_s = m_s + m_D, \tag{4}$$

respectively.

The SQM free energy contributed by the Fermi gas reads [9]

$$
\begin{aligned}
\Omega(\mu_i, M_i, p_F^i) &= \sum_{i=u,d,s} \int_0^{p_F^i} \frac{3}{\pi^2} p^2 (\sqrt{p^2 + M_i^2} - \mu_i) dp \\
&= -\sum_{i=u,d,s} \frac{1}{4\pi^2} [\mu_i p_F^i (\mu_i^2 - \frac{5}{2} M_i^2) + \frac{3}{2} M_i^4 \ln(\frac{\mu_i + p_F^i}{M_i})].
\end{aligned} \tag{5}
$$

For each flavor the Fermi momentum p_F^i is defined by $p_F^i = \sqrt{\mu_i^2 - M_i^2}$ where $\mu_u = \mu - 2\mu_e/3$ and $\mu_d = \mu_s = \mu + \mu_e/3$ if the electron chemical potential $\mu_e \neq 0$. On the SQM side, the Fermi momenta of u, d and s quarks are different and are related to the corresponding densities via $\rho_i = (p_F^i)^3/\pi^2$. Therefore, for SQM, the electrical neutrality is realized by

$$\frac{2}{3}\rho_u - \frac{1}{3}(\rho_d + \rho_s) = \rho_e = \frac{\mu_e^3}{3\pi^2}, \tag{6}$$

and the baryon density is

$$\rho = \frac{1}{3}(\rho_u + \rho_d + \rho_s). \tag{7}$$

When the contribution from electrons is included, the total free energy for SQM becomes

$$\Omega_{SQM} = \Omega(\mu_i, M_i, p_F^i) - \frac{\mu_e^4}{12\pi^2}. \tag{8}$$

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With respect to nuclear matter, SQM becomes energetically stable for low densities as long as its energy per baryon satisfies

$$(\mathcal{E}/\rho)_{SQM} \leq 930\text{MeV}, \tag{9}$$

at zero pressure, where 930MeV corresponds to a typical value of the energy per baryon in nuclei. For our purpose, Eq.(9) needs to be considered seriously to guarantee that not nuclear matter but SQM undergoes a phase transition to CFL (if without this constraint the nuclear-CFL transition [6] would be very likely). In this work, we fix the parameter C by the critical condition of Eq.(9) for certainty. For instance, the value of C is adopted to be 110 and 70MeV/fm^3 as $m_s = 0$ and 180MeV respectively (see Ref.[9] for details).

On the other hand, the energy per baryon for two-flavor quark matter (2QM) is required to satisfy the inequality

$$(\mathcal{E}/\rho)_{2QM} > 930\text{MeV}, \tag{10}$$

at zero pressure [2]. By using Eqs.(9) and (10), the stability window can be obtained in which SQM corresponds to the stable ground state at low densities. But Eq.(10) does not apply to the moderate-density case. Due to the appearance of strange flavor, SQM is favored over the regular two-flavor matter as long as

$$\mu_d = \mu + \mu_e/3 \geq M_s. \tag{11}$$

Instead of Eq.(10), thus, Eq.(11) needs to be taken into account in the following calculation.

B. CFL and gCFL in the model

Different from the unpaired one, the CFL quark matter is an insulator in which no electrons are required for the electrical neutrality [11]. The Fermi momenta have the common value [6, 11]

$$\nu = 2\mu - \sqrt{\mu^2 + m_s^2/3}, \tag{12}$$

for all three flavors and μ_e does not influence the CFL free energy directly [2] . Thus the CFL free energy contributed by the Fermi gas is obtained by replacing the variables μ_i and p_F^i in Eq.(5) by μ and ν respectively. Together with the contribution from the CFL pairing (the third term in Eq.(1)), the total free energy for CFL takes the form

$$\Omega_{CFL} = \Omega(\mu, M_i, \nu) - \frac{3\Delta^2\mu^2}{\pi^2}, \tag{13}$$

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[2]Assuming that the SQM-CFL transition is of first order, there is an interface between the electron-rich SQM and the electron-free CFL. In this case, the effective value of electron chemical potential is zero on the CFL side because of the electrostatic potential at the metal-insulator boundary (see Refs.[6, 12] for details). In the present work, a possibility of the mixed state consisting of the electrical-opposite SQM and CFL will be ignored. The reason is that the nonzero m_D provides the additional instanton interaction so that the electrical-negative phase such as CFLK^- is very difficult to emerge.

when the density dependence of quark mass is considered.

At high density, the value of m_D is close to zero so that the difference between Eqs.(1) and (13) becomes negligible. For the concerned density region, Eq.(13) means that not only m_s but also m_D contribute to the CFL free energy. As a consequence, the previous results of the SQM-CFL transition e.g. Fig. 1 might be no longer valid at low/moderate densities (see Sec.III for details).

Now we turn to consider the gapless color-flavor-locked (gCFL) phase where the Cooper pairs between the blue-down (bd) and green-strange (gs) quasi quarks becomes unstable. At sufficiently high densities, the strange quark mass is just the current mass m_s and the light quark masses are very small so that the mass matrix reduces to $diag(0,0,m_s)$ approximately. It is well known that the quark mass term can be simplified as

$$-\frac{m_s^2}{2\mu}(\tilde{\psi}_L^{s\,+}\tilde{\psi}_L^s + L \to R), \tag{14}$$

at the leading order of the CFL effective Lagrangian [13, 14]. Here $\tilde{\psi}$ is not the ordinary quark field, but the quasi-quark field in the vicinity of the Fermi surface. For the bd-gs Cooper pairs, $\tilde{\psi}_{L/R}^s$ in Eq.(14) denotes the gs modes near the Fermi surface. It is worth being notified that Eq.(14) conserves chiral symmetry although the current mass term in the low-density QCD theory does not. As suggested in Ref.[14], $m_s^2/(2\mu)$ is regarded as the chemical potential associated with strangeness, i.e. $\mu_S = m_s^2/(2\mu)$. Obviously, the effective chemical potential for the gs modes is influenced by the nonzero μ_S while that for the bd modes is independent of μ_S. As a result, the effective chemical potentials μ_{gs}^{eff} and μ_{bd}^{eff} become different. To account for the relative chemical potential of the paired bd and gs modes, the variation is [8]

$$\delta\mu = \frac{\mu_{bd}^{eff} - \mu_{gs}^{eff}}{2} = \frac{m_s^2}{2\mu}, \tag{15}$$

where the contribution from the chemical potential associated with the color charge has been included. When the variation is larger than the color superconducting gap, i.e.

$$\frac{m_s^2}{2\mu} \geq \Delta, \tag{16}$$

gCFL emerges as the more stable phase than CFL. Based on Eq.(16), gCFL was predicted to emerge at relatively large m_s and/or relatively small μ, as shown in Fig. 1.

However, this is not the whole story yet. At densities where m_D becomes nonzero, the variation $\delta\mu$ caused by the strange mass term mainly is influenced by m_D also. This means that the gCFL existence at moderate densities must be reexamined once the density dependence of quark mass is considered. Noticing that m_D is the dynamical mass essentially, it does not enter the CFL effective Lagrangian via a simple replacement $m_s \to M_s = m_s + m_D$ directly. Therefore, an extrapolation like $\delta\mu \to \delta\mu' = \frac{M_s^2}{2\mu} = \frac{(m_s+m_D)^2}{2\mu}$ is not feasible in principle.

To incorporate the effect of m_D self-consistently, let us consider the dynamical quark mass term in the Lagrangian involving the quasi-quark degrees of freedom. Note that the chiral symmetry pattern exhibits a resemblance to the color-flavor-locking pattern. It is reasonable to assume that m_D enters the CFL effective Lagrangian via the chiral-invariant form

$$\xi m_D({\tilde{\psi}_L^s}^+ \tilde{\psi}_L^s + L \to R). \tag{17}$$

Without losing generality, we introduce an unknown coefficient ξ in Eq.(17). If the density-dependent-mass m_D does account for all the non-perturbative effects of the dynamical chiral breaking, ξ is equal to 1 which is adopted in the calculations of Sec.III. Eq.(17) is expected to deviate the strangeness chemical potential from the original value $\mu_S = m_s^2/(2\mu)$. Correspondingly, only the effective chemical potential μ_{gs}^{eff} is influenced by Eq.(17) for the bd-gs Cooper pairs. In analogy with the treatment of yielding Eq.(15), there exists a decrease in the value of $\delta\mu$ so that we redefine the variation as

$$\delta\mu \to \delta\mu' = \frac{m_s^2}{2\mu} - \frac{\xi}{2}m_D. \tag{18}$$

By inserting Eq.(18) into the dispersion relation for the bd-gs pairs [15], the gapless modes become possible at the momenta

$$p_{gapless}^{\pm} = \overline{\mu} \pm \sqrt{(\delta\mu')^2 - \Delta^2}, \tag{19}$$

where $\overline{\mu}$ is the average value of the bd and gs effective chemical potentials. Compared with the case without m_D, the " blocking " region [16], i.e. the width between $p_{gapless}^+$ and $p_{gapless}^-$, is suppressed for not-very-high densities. As a consequence, the free energy contributed by the gapless phenomenon is expected to be influenced by both m_s and m_D since the relative free energy of gCFL and CFL depends on the magnitude of the " blocking " region mainly.

Based on Eq.(18), the condition Eq.(16) is replaced by

$$\frac{m_s^2}{2\mu} - \frac{\xi}{2}m_D \geq \Delta. \tag{20}$$

If CFL emerges at the densities where m_D becomes negligible, Eq.(20) reduces back to Eq.(16) and the critical condition for the CFL-gCFL transition is still $m_s^2/(2\mu) = \Delta$ approximately. Also, the necessary condition Eq.(2) for the existence of the CFL matter needs to be reexamined in the case of $m_D \neq 0$. Once the variation between the bd and gs chemical potentials is too large, the color-flavor-locked pairing might be broken completely. According to Ref.[16], the CFL matter including gCFL exists only when the variation is not larger than 2Δ which is the energy cost for breaking a Cooper pair. So the previous condition Eq.(2) is extended to

$$\frac{m_s^2}{2\mu} - \frac{\xi}{2}m_D \leq 2\Delta. \tag{21}$$

For the moderate density region, Eq.(21) provides a more realistic boundary for the CFL matter.

III. NUMERICAL RESULTS AND CONCLUSIONS

As a strong-coupling effect, the nonzero m_D is expected to affect the phase diagram of three-flavor quark matter for not-very-high densities. Before going to be specific, let us firstly discuss *whether or not* the phase transition between SQM and CFL/gCFL occur in the moderate density region. The answer is not always positive and it is actually linked to the value of m_s, as argued in the following.

Now both SQM and CFL/gCFL are the deconfined phases, therefore the physics of confinement does not play a role in determining the SQM-CFL/gCFL transitions. So the negative values of the free energies obtained in Sec. II are related to the corresponding pressure directly and then the Gibbs condition for the pressure equilibrium reads

$$P_{CFL/gCFL} - P_{SQM} = \Omega_{SQM} - \Omega_{CFL/gCFL} = 0. \tag{22}$$

For $m_s = 10$, 50, 100 and 150MeV, we show $\delta P = P_{CFL} - P_{SQM}$ as a function of $1/\mu$ in Fig. 2. It is found that δP does no longer approach to zero monotonously with increasing $1/\mu$, i.e. decreasing μ. As shown in Fig. 2, there exists a rising tendency of δP in the vicinity of $\mu \simeq 0.3$GeV. This leads to the fact that no any pressure equilibrium appears in the moderate density region so that a first-order SQM-CFL phase transition does not occur. Although the CFL pressure is relatively large, the absence of the phase transition means that the CFL matter is impossible to exist at least in our concerned density region. Therefore, SQM with small m_s still remains as a stable state for moderate densities while CFL with small m_s is prohibited unless the density is very large [3].

The above physical picture holds valid until m_s is large. For larger m_s, more pressure is paid to maintain the common Fermi momentum so that the pressure of CFL decreases. As long as m_s is large enough, the SQM-CFL transition in moderate density region becomes possible to occur. Our numerical calculation shows that, as m_s is about 150MeV, the pressure equilibrium comes to appear in the vicinity of $\mu \simeq 0.3$GeV (see Fig. 2 also). Noticing that such pressure equilibrium behaves like " crossover " of CFL and SQM, $m_s \simeq 150$MeV is regarded as the minimal value allowed for the CFL existence at moderate density. Once m_s is larger than 150MeV, the transition from SQM to CFL/gCFL might occur in moderate density region.

As a typical example, the result of $\delta P = P_{CFL} - P_{SQM}$ for $m_s = 200$MeV is given by the solid line in Fig. 3. As shown in Fig. 3, the critical chemical potential μ_c for the SQM-CFL transition is about 0.4GeV. When the gCFL phase and the SQM-gCFL transition are incorporated, however, the SQM-CFL transition might not occur

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[3]At a very high density, the pressures for SQM and CFL can become close to each other as long as the pairing gap Δ is small enough compared with the value of μ. In the asymptotic sense, the SQM-CFL transition always occur regardless of whether m_s is small. But this is not the case being concerned in the present work.

in nature since gCFL is more energetically favorable than CFL. The dashed line in Fig. 3 gives the value of $P_{gCFL} - P_{SQM}$ for $m_s = 200$MeV. As shown in Fig. 3, the SQM-gCFL transition occurs at the critical chemical potential $\mu'_c \simeq 0.35$GeV, which is smaller than the value of μ_c. Therefore, it is not CFL but gCFL to emerge firstly in the quark star environment. Also, the critical point for the gCFL-CFL transition μ''_c is shown to be about 0.67GeV in Fig. 3. For $m_s = 200$MeV, we conclude that, gCFL exists as a stable state in the region of $\mu'_c < \mu \leq \mu''_c$ and CFL emerges as $\mu > \mu''_c$. Here we would like to emphasize the importance of the nonzero m_D for the properties of gCFL. In Eq.(18) the $m_s^2/(2\mu)$ and m_D terms provide opposite contributions to the gCFL pressure actually. Within the framework where the density dependence of quark mass is considered, if the gapless phenomenon were determined by $m_s^2/(2\mu)$ exclusively the pressure of gCFL would be much larger than that of SQM with increasing $1/\mu$, as shown by the dotted line in Fig. 3. In that case, the SQM-gCFL pressure equilibrium and thus the corresponding transition no longer exists in the moderate density region. Therefore the effect of m_D is relevant to the gCFL presence for moderate densities.

Based on the above arguments, a schematic phase diagram is given for the moderate density region in Fig. 4. There are three kinds of different structures in the phase diagram according to the value of m_s :

(i) As m_s is small such as $m_s < 150$MeV , the effect of m_D prohibits the CFL formation for not-very-high densities. In this case, it is not CFL but SQM to be the stable phase in the whole moderate density region, as shown in Fig. 4. This conclusion is very different from that obtained by Fig. 1, but agrees with the original prediction that CFL with zero (or small) m_s becomes possible only when the density is high enough [4]. When m_s is slightly larger than 150MeV, a first-order transition from SQM to CFL takes place. For instance, let us consider the horizontal line of $m_s = 175$MeV in Fig. 4. At the critical chemical potential $\mu_c \simeq 0.35$GeV, the SQM-CFL transition occurs so that SQM remains stable for $\mu < \mu_c$ whereas CFL emerges for $\mu \geq \mu_c$.

(ii) As m_s is large, the gCFL phase becomes more likely than the conventional CFL phase and then the SQM-gCFL transition replaces the SQM-CFL transition to be relevant to the phase diagram. Considering the horizontal line of $m_s = 200$MeV, gCFL comes to emerge at $\mu'_c \simeq 0.35$GeV and it undergoes a continuous transition to CFL at $\mu''_c \simeq 0.67$GeV. Interestingly, the SQM-gCFL transition curve in the low density region is qualitatively different from the result of Fig. 1 : the critical value of m_s does increase with decreasing μ, as shown by the solid line in Fig. 4. The reason is that the gapless phase is determined by Eq.(18), in which the $m_s^2/(2\mu)$ and m_D terms offer opposite effects on the gCFL pressure. For lower μ the latter effect is more important, so that the transition becomes possible only when m_s is relatively large. Also, the continuous gCFL-CFL transition curve is given by Eq.(20) as shown by the dashed line in Fig. 4. Together with the SQM-CFL and SQM-gCFL transition curves, there exists an intersection of the three phase transitions in the vicinity of (m_s, μ)= (185MeV,0.4GeV).

(iii) As m_s is larger, the variation $\delta\mu'$ might be too large to allow existence of the color-flavor-locked pairing in quark matter. Most importantly, the necessary condition

Eq.(21) for existence of the CFL matter no longer coincides with the SQM-gCFL transition curve, although Eq.(2) does in the case without m_D. Thus, the gCFL phase region is surrounded by the CFL boundary curve (that obtained from Eq.(21)), the SQM-gCFL transition curve (that determined by the pressure equilibrium) and the gCFL-CFL transition curve (that obtained from Eq.(20)), as shown in Fig. 4. Comparing with Fig. 1, we find that the gCFL phase region is suppressed for low μ. On the other hand, the existence of three-flavor quark matter might be prohibited if m_s is very large. The boundary curve of SQM is given by Eq.(11) and is shown in Fig. 4, which seems to imply that 2QM is irrelevant to the gCFL presence. At this point, it must be stressed that a possibility of the 2QM-gCFL transition could not be ruled out simply if the color superconductivity exists in the two-flavor quark matter. In that case, the boundary of SQM shown in Fig. 4 needs to be modified also. Details involving such 2QM and its transition to gCFL and/or SQM depend on what kind of two-flavor color superconducting phase be taken into account, which is beyond the scope of the present work.

In summary, we extend the descriptions of CFL and gCFL from high-density case to the moderate density region when the density dependence of quark mass is considered. Starting at low density and raising the matter density, the physical picture that SQM remains as the stable state at first and then undergoes a first-order phase transition to gCFL and/or CFL is examined in details. As a result, we predict a more complicated phase diagram of three-flavor quark matter, in which the CFL/gCFL phase region is suppressed for low densities. The present phase diagram is helpful to better understand the ground state of three-flavor quark matter in the environment of quark stars. Of course, there are some uncertainties of the color superconducting gap used in this work. When the value of the gap is chosen in other ways, we can give the similar phase diagram as Fig. 4. For instance, if the gap is large such as $\Delta \sim 80\text{MeV}$ [17] we find that the minimum value of m_s allowed for the CFL existence increases so that the CFL phase region for moderate densities might be further suppressed. Even if the density dependence of the gap is included, the change of Δ in the finite region of $\mu = 0.3 - 1\text{GeV}$ is not too drastic and the conclusion obtained from Fig. 4 is still qualitatively correct. In the further work one should construct the dynamical quark mass within a more realistic framework such as that beyond the bag model as well as take the contributions from the color-sextet pairing and the gap equation into account. Some of the problems are being investigated.

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Figure 1: Schematic phase diagram in the (m_s,μ) plane, where the solid line is the boundary of the CFL matter obtained from Eq.(2) and the dashed line is the phase transition from the conventional CFL phase to the gCFL phase obtained from Eq.(16) (see the following).
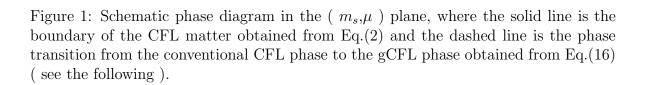
Figure 2: The CFL pressure vs. the SQM pressure. The solid lines from top to bottom are the relative pressures for $m_s = 10, 50, 100$ and 150MeV respectively.

Figure 3: The CFL/gCFL pressures vs. the SQM pressure for $m_s = 200$MeV, where ξ is chosen to be 1 for simplicity. The solid (dashed) line is the relative pressures for CFL (gCFL), while the dotted line is that for gCFL which is determined by Eq.(15) exclusively.

Figure 4: Similar as Fig. 1 but the SQM-CFL/gCFL transitions are considered in the case of including effects of m_D. The solid lines are the first order SQM-CFL/gCFL transitions, the dashed line is the continuous CFL-gCFL transition and the dot-dashed lines correspond to the boundaries of two kinds of three-flavor quark matter.